CBCS SCHEME

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15MAT31

Third Semester B.E. Degree Examination, July/August 2021
Engineering Mathematics – III

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions.

1 a. Obtain the Fourier series for the function,

$$f(x)=\begin{cases}1+\dfrac{2x}{\pi} & -\pi\le x\le 0\\ 1-\dfrac{2x}{\pi} & 0\le x\le \pi\end{cases}$$

Hence deduce that $\dfrac{1}{1^2}+\dfrac{1}{3^2}+\dfrac{1}{5^2}+.....=\dfrac{\pi^2}{8}$ **(08 Marks)**

b. Find the constant term and first two harmonics in the Fourier series for f(x) given by the following table:

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
f(x)	1.0	1.4	1.9	1.7	1.5	1.2	1.0

(08 Marks)

2 a. Expand $f(x)=\sqrt{1-\cos x}$ in $0\le x\le 2\pi$ in a Fourier series. Evaluate $\dfrac{1}{1.3}+\dfrac{1}{3.5}+\dfrac{1}{5.7}+.....$ **(08 Marks)**

b. Obtain the Fourier series for f(x) = |x| in $(-\ell, \ell)$ and hence evaluate $\dfrac{1}{1^2}+\dfrac{1}{3^2}+\dfrac{1}{5^2}+.....=\dfrac{\pi^2}{8}$ **(08 Marks)**

3 a. Find the Fourier transform of $f(x)=\begin{cases}1-|x| & \text{for} & |x|\le 1\\ 0 & \text{for} & |x|>1\end{cases}$ and hence deduce that $\displaystyle\int_0^\infty \frac{\sin^2 t}{t^2}dt$ **(06 Marks)**

b. Find the Fourier sine transform of $e^{-|x|}$. Hence show that $\displaystyle\int_0^\infty \frac{x\sin mx}{1+x^2}dx=\frac{\pi}{2}e^{-m}$ where $m>0$. **(05 Marks)**

c. Find the z-transform of (i) $(2n-1)^2$ (ii) $\cos\left(\dfrac{n\pi}{2}+\dfrac{\pi}{4}\right)$ **(05 Marks)**

4 a. Find the Fourier transform of $f(n)=\begin{cases}1 & |x|\le 1\\ 0 & |x|>a\end{cases}$. Hence deduce $\displaystyle\int_0^\infty \frac{\sin ax}{x}dx$. **(06 Marks)**

b. Find the inverse z-transform of $\dfrac{4z^2-2z}{z^3-5z^2+8z-4}$. **(05 Marks)**

c. Solve the differential equation $u_{n+2}+6u_{n+1}+9u_n=2^n$ with $u_0 = u_1 = 0$ using z-transform method. **(05 Marks)**

5 a. Find the coefficient of correlation and the two lines of regression for the following data:

x	1	3	4	2	5	8	9	10	13	15
y	8	6	10	8	12	16	16	10	32	32

(06 Marks)

b. Fit a curve of the form $y = ae^{bx}$ to the following data:

x	77	100	185	239	285
y	2.4	3.4	7.0	11.1	19.6

(05 Marks)

c. Use Regula Falsi method, find the root of the equation $x^2 - \log_e x - 12 = 0$. (05 Marks)

6 a. The two regression equations of the variables x and y are x = 19.13 – 0.87y and y = 11.64 – 0.5x. Find:
(i) Means of x
(ii) Means of y
(iii) The correlation coefficient (06 Marks)

b. Fit a parabola $y = a + bx + cx^2$ to the following data:

x	−3	−2	−1	0	1	2	3
y	4.63	2.11	0.67	0.09	0.63	2.15	4.58

(05 Marks)

c. Use Newton-Raphson method to find the real root of $3x = \cos x + 1$, take $x_0 = 0.6$ perform 2 iterations. (05 Marks)

7 a. Find the cubic polynomial by using Newton forward interpolating formula which takes the following values.

x	0	1	2	3
y	1	2	1	10

(06 Marks)

b. Apply Lagrange's formula inversely to obtain a root of the equation f(x) = 0 given that f(30) = −30, f(34) = −13, f(38) = 3, f(42) = 18. (05 Marks)

c. Use Weddle's rule to evaluate $\int_0^{\pi/2} \sqrt{\cos\theta}\, d\theta$ dividing the interval $\left[0, \frac{\pi}{2}\right]$ into six equal parts. (05 Marks)

8 a. A survey conducted in a slum locality reveals the following interpolating information as classified below:

Income/day in rupees : x	Under 10	10-20	20-30	30-40	40-50
Number of persons : y	20	45	115	210	115

Estimate the probable number of persons in the income group 20 to 25. (06 Marks)

b. Using Newton divided difference formula fit an interpolating polynomial for the following data:

x	0	1	4	5
f(x)	8	11	68	123

(05 Marks)

c. Using Simpson's 1/3rd rule evaluate $\int_0^1 \frac{x^2}{1+x^3}dx$ taking four equal strips. (05 Marks)

9 a. Find the extremal of the functional $I = \int_0^{\pi/2} (y^2 - y'^2 - 2y\sin x)dx$ under the conditions $y(0) = y\left(\frac{\pi}{2}\right) = 0$. **(06 Marks)**

b. If $\vec{F} = x^2 i + xyj$ evaluate $\int_C \vec{F}.d\vec{r}$ from (0, 0) to (1, 1) along

(i) the line $y = x$ (ii) the parabola $y = \sqrt{x}$ **(05 Marks)**

c. Find the curve passing through the points (x_1, y_1) and (x_2, y_2) which when rotated about the x-axis gives a minimum surface area. **(05 Marks)**

10 a. Verify Green's theorem in a plane for $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where c is the boundary of the region enclosed by $y = \sqrt{x}$ and $y = x^2$. **(06 Marks)**

b. Using divergence theorem evaluate $\int \vec{A} \cdot \hat{n}\, ds$ where $\vec{A} = x^3 i + y^3 j + z^3 k$ and s is the surface of the surface $x^2 + y^2 + z^2 = a^2$. **(05 Marks)**

c. Find the geodesics on a surface given that the arc length on the surface is $s = \int_{x_1}^{x_2} \sqrt{x(1 + y'^2)}dx$. **(05 Marks)**

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